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Time Series Analysis & Forecasting

19.1 INTRODUCTION

From economic situation, if it relates to an individual firm, industry or a nation as a whole, we can observe a continuous movement of economic activity. In order to describe this flow of economic activity, the statistician uses a time series. The term 'Time Series' means a set of observations concurring any activity against different periods of time. The duration of time period may be hourly, daily, weekly, monthly or annually.

Following are few examples of time series data:

- a) Profits earned by a company for each of the past five years.
- b) Workers employed by a company for each of the past 15 years.
- c) Number of students registered for CA examination in the institute for the past five years.
- d) The weekly wholesale price index for each of the past 30 week.
- e) Number of fatal road accidents in Delhi for each day for the past two months.

19.2 IMPORTANCE OF TIME SERIES ANALYSIS

There are several reasons for undertaking a time series analysis. Firstly, the analysis of a time series helps to understand the past performance. Secondly, a time series analysis helps directly in business planning. A firm can know the long-term trend in the sale of its product.

Thirdly, a time series analysis helps one to study such movement as cycles that fluctuates in two or more series regarding the rate or type of growth.

From the above discussion, we can conclude that time series analysis has great advantages in business and industry.



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19.3 COMPONENTS OF A TIME SERIES

A time series may contain one or more of the following four components:

1. Secular trend (T)
2. Seasonal variation (S)
3. Cyclical variation (C)
4. Irregular variation (I)

There are two approaches for the relationship amongst these components.

(a) $Y = T \times S \times C \times I$ (multiplicative model)

(b) $Y = T + S + C + I$ (additive model)

Where Y is the result of the four components.

The effects of these components might be multiplicative or additive or might be a combination of several other form. On account of this, different assumptions will give different results. However, multiplicative components is most frequently used.

19.3.1 TREND

The trend is the long-term movement of a time series. Any increase or decrease in the values of a variable occurring over a period of several years gives a trend. If the values of a variables remain statutory over several years, then no trend can be observed in the time series. To study the growth in industrial production from the year 1995 to 2005, we need to find the trend values in industrial production for this time period which may be increasing or decreasing.

These trends may be either linear or non-linear. There are other types of trends like parabolic (or quadratic) and logarithmic (or exponential). However we are more concerned here with straight line (i.e. linear) trends.

The various methods of fitting a straight line to a time series such as free hand method, the method of semi-averages, the method of moving averages and the method of least squares.

(a) The Freehand Method: It is the simplest method of finding a trend line. The procedure involves first the plotting of the time series on a graph and fitting a straight line through the plotted points in such a way that the straight line shows the trends of the series.

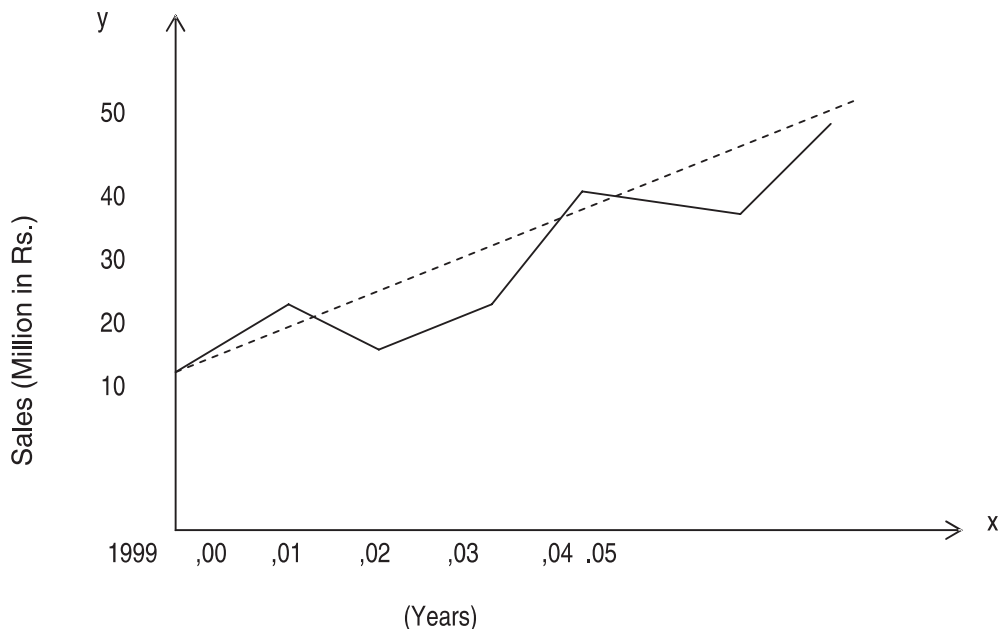
e.g. Sales of products 'A' 1999-2005

<u>Year (x)</u>	<u>Sales (y) (Rs. in Million)</u>
1999	10
2000	20
2001	15
2002	25
2003	37
2004	35
2005	40



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Taking X axis as time (in years) and y axis to the sale of product in million rupees, sales figures are plotted and joining them by straight line we get fluctuating straight lines through which we get an average straight line. It may be noted that the free hand method is not an accurate method of filling a trend as different persons may fit different trend times to the same set of data.



(b) **The Method of Semi-averages:** When the method of semi-averages is used, the given time series is divided into two parts preferably with the same number of years. The average of each part is calculated and then a trend line through these average is filled.

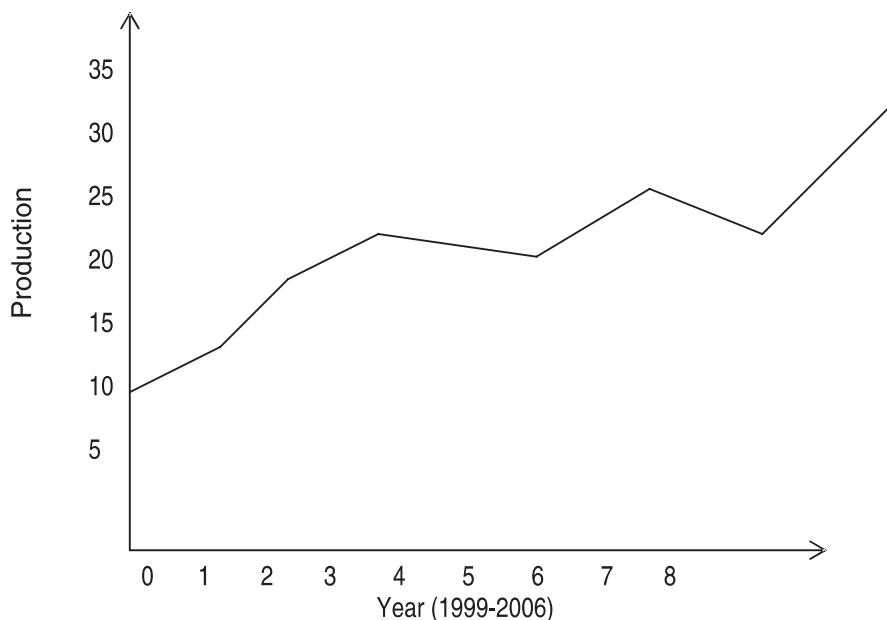
Production from 1999-2006

<u>Year</u>	<u>X</u>	<u>Y</u>	<u>Average</u>
1999	0	10	60/4 = 15
2000	1	12	
2001	2	18	
2002	3	20	100/4 = 25
2003	4	20	
2004	5	25	
2005	6	23	
2006	7	32	



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The average of first part of the data is 15 and that of the second part is 25. As 15 is the average of 1999, 2000, 2001 and 2002. 15 is plotted in between 2000 and 2001 which is the middle of the 4 years period. Similarly 25 is plotted in between 2004 and 2005. Then these points are joined by a straight line which is a semi-average trend line.



(c) **The method of moving average:** The method of moving averages is used not only to fit trend lines but also to seasonal and cyclical variation.

e.g. Find out the three year moving averages starting from 1989.

Year:	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
2000											
Sales:	10	15	20	25	15	12	15	24	15	21	15
				24							

(Rs. in million)

Solution: Taking the first three years 1989, 1990 and 1991, we add up their sales $10 + 15 + 20 = 45$. This figure is written in column (3) against the mid year that is 1990. Now dropping the year 1989 and including the year 1992 in the calculation and thus moving in this manner the process is continued until we reach the last three years 1998, 1999 and 2000. It may be noted that there are no moving total for the 1st and last year.



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The next step involves the calculation of moving average by dividing the moving total by 3.

Table:

Year	Sales Rs. in Million	3 year moving total	3 year moving average
1989	10	—	—
1990	15	45	15
1991	20	60	20
1992	25	60	20
1993	15	52	17
1994	12	42	14
1995	15	51	17
1996	24	54	18
1997	15	60	20
1998	21	54	18
1999	18	63	21
2000	24	—	—

Effective Application of Moving Average Method:

To ensure result of moving average method to be appropriate and effective, it is required to ascertain first whether a regular periodic cycle in the time series exists. In several cases one would find that there is a certain regularity in the series to allow the use of the moving average method. It may be also noted that if the basis nature of the time series is linear, it will give a linear trend. In case of curvilinear nature, the trend will be curve. Moreover moving average method helps to element seasonal fluctuation, for a time series.

Method of Least squares: Among the method of fitting straight line to a series of data, this method is the most frequently used method. The equation of a straight line is $Y = a + bX$ where X is the time period, say, year and Y is the value of the item measured against time, a is the Y- intercept and b is the coefficient of X indicating slope of the trend line.

In order to find a and b the following two equations are solved:

$$\Sigma Y = ax + b \Sigma x$$

$$\Sigma XY = a \Sigma x + b \Sigma x^2$$

Where n is the total number of observations in a series. These equations are called normal equations.

e.g. Old number of years:

Year:	1996	1997	1998	1999	2000
Y	270	285	295	315	330



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Fit a linear trend to these data using method of least squares:

Solution:

Year	x	y	xy	x ²
1996	-2	270	-540	4
1997	-1	285	-285	1
1998	0	295	0	0
1999	1	315	315	1
2000	2	330	-660	4
Total	0 = Σx	1,495 = Σy	150 = Σxy	10 = Σx^2

Substituting these values in two normal equations.

We get

$$1495 = 5a + b(0) \dots\dots\dots(1)$$

$$150 = a(0) + b(10) \dots\dots\dots(2)$$

Equations (1) gives $a = 1,495/5 = 299$

Equations (2) gives $b = 150/10 = 15$

The straight line trend we get is

$$Y = 299 + 15x \quad (\text{origin: year 1998})$$

$$(x = 1\text{- year unit})$$

e.g. **Even Number of years:**

Year:	1985	1986	1987	1988	1989	1990
Y	15	14	18	20	17	24

Fit a trend to these data using method of least squares.

Solution

Year	x	y	xy	x ²
1985	-5	15	-75	25
1986	-3	14	-42	9
1987	-1	18	-18	1
1988	1	20	20	1
1989	3	17	51	9
1990	5	24	120	25
Total	0 = Σx	108 = Σy	56 = Σxy	70 = Σx^2



Two normal equations are:

$$\Sigma Y = an +b \Sigma x$$

$$\Sigma XY= a\Sigma x +b\Sigma x^2$$

Substituting the values in the two equations, we get

$$108 = 6a +b (0) \dots\dots\dots(1)$$

$$56 = a(0) +b (70)\dots\dots\dots(2)$$

Solving we get $a = 108/6 = 18$

And $b = 56/70 = 0.8$

The trend equation is $Y = 18 +0.8 X$

Origin year: Mid of 1987-88

Unit of X = 1/2 year

Note:

- (i) Putting values of x for different years we get values of y which are trend values (y)
- (ii) For forecasting, to calculate the value of x, for a given future year and substituting the value of x in the trend equation to get the forecasted value.

Higher Degree Polynomial Trends:

So far we have discussed linear trend which in fact is a polynomial trend of the first degree. However instead of straight line trend, some parabolic trend may be best fit when we have to use second degree polynomial trend. This trend equation is

$$Y = a+bx+cx^2$$

In this also method of least sequence is to be used where there are normal equations

$$\Sigma Y = na +b \Sigma x + c \Sigma x^2$$

$$\Sigma xy = a \Sigma x +b \Sigma x^2 +c \Sigma x^3$$

$$\Sigma x^2y = a\Sigma x^2 +b\Sigma x^3 +c \Sigma x^4$$

e.g. fit quadratic trend:

Year :	1995	1996	1997	1998	1999
Production Of cars:	2	4	8	14	22
(‘0000)					

Solution

Year	x	x ²	y	xy	x ² y	x ³	x ⁴
1995	-2	4	2	-4	8	-8	16
1996	-1	1	4	-4	4	-1	1
1997	0	0	8	0	0	0	0
1998	1	1	14	14	14	1	1
1999	2	4	22	44	88	8	16
Total	0	10	50	50	114	0	34



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Applying these values in the above equations, we get

$$50 = 5a + 10c \dots\dots\dots(1)$$

$$50 = 10b \dots\dots\dots(2)$$

$$114 = 10a + 34c \dots\dots\dots(3)$$

From (2) we find $b = 50/10 = 5$

Again, $50 = 5a + 10c \dots\dots\dots(1)$

$$114 = 10a + 34c \dots\dots\dots(2)$$

$$(3) \quad 114 = 10a + 34c$$

$$(1) \times 2 \quad \underline{100 = 10a + 20c}$$

$$14 = 14c$$

$$\therefore c = 1$$

From (1), $50 = 5a + 10$ $\therefore a = 8$

Required trend equation is

$$Y = a + bx + cx^2$$

$$Y = 8 + 5x + x^2 \quad \text{origin year 1997}$$

Unit of $x = 1$ year

Exercise

(1) Assuming an additive model, obtain 3 year-moving average to obtain the trend free series for year 1992 to 1996 from the following data:

Year:	1991	1992	1993	1994	1995	1996	1997
Export:	126	130	137	141	145	155	159

(Rs. in lakhs)

Answer: 131, 136, 141, 147, 153

(2) Use a 4 yearly moving average method to calculate trend for the following data:

Year:	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988
Production:	614	615	652	678	681	655	717	719	708	779	757

Answer: ____, ____, 648, 661.4, 674.63, 687.9, 690.1, 715.3, 735.8, ____, ____.

(3) Give below are the figures of production (in million tons) of wheat

Year:	1989	1990	1991	1992	1993	1994	1995
Production:	80	90	92	83	94	99	92

Fit a straight line trend to these figures.

Answer: $Y = 90 + 2x$, origin year 1992, x unit = 1 year]



(4) Fit a straight line by the least squares method to the following figures of production of Sugar factory:

Year	1979	1980	1981	1982	1983	1984	1985
Production	76	87	95	81	91	96	90

Estimate the production for 1986.

Answer: $Y=88+2x$; 96 ('000 tons)

(5) The following table given the annual profit (in thousand Rs.) in a factory

Year:	1991	1992	1993	1994	1995	1996	1997
Profit	60	72	75	65	80	85	90

Fit a straight line trend by the method of least squares. Also find annual trend of the profit. Also calculate the projected profit for 1998.

Answer: $Y = 75.286 + 4.321x$, 4.321, 92.57 ('000 Rs.)

(6) Below are given the annual consumption in thousand tons of a town

Year	1985	1986	1987	1988	1989	1990	1991
Consumption	70	75	90	91	95	98	100

('000 tons)

Fit a straight line trend by the method at least square and indicate the trend values by taking 1988 as origin. Shift the origin to 1990 and find out the revised trend equation.

Construct the annual trend equation into monthly trend equation.

Answer. $Y = 88.43 + 5.04x$, $Y = 98.51 + 5.05x$, $Y = 7.37 + 0.035 x$]

(7) Fit a quadratic trend to the following data:

Year	1988	1989	1990	1991	1992
Value	180	192	195	204	202

Answer: $Y = 197.74 + 5.6x - 1.57x^2$]

Detrending: The process of eliminating the trend is referred to as detrending and the trend itself can be represented either as a straight line or some type of smooth curve. What is left after elimination of trend is short term variation, expressed as:

$S \times C \times I$ Under multiplicative model,

And $S + C + I$ Under additive model.

19.3.2 The Calculation of Seasonal Variations

These are short-term movements which recur after a period. The period is normally less than a year-it may be a month or a quarter of the year. For example, the prices of grain vary between the harvest season and the non-harvest season.



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For purpose of time series analysis we isolate the seasonal element. There are two major reasons for this : (i) to study seasonal variation; and (ii) to eliminate them.

- I. Seasonal Variations in themselves give a clear idea about the relative position of each season on the basis of which it is possible to plan for the season.
- II. It is necessary to eliminate seasonal factor to get the cyclical or irregular factors. The is called deseasonalization of the data.

First we take an illustration of isolating short-term and seasonal fluctuations by the additive model.

Illustration:

Year	Quarter Q	Output Y	Moving total (4)	Moving total (8)	Moving Average (T)	Short-term Fluctuation $Y-T=S+C+I$
1971	1	35				
	2	86	312			
	3	67	315	627	78	-11
	4	124	338	653	82	+42
1972	1	38	362	700	88	-50
	2	109	414	776	97	+12
	3	91	423	837	105	-14
	4	176	472	895	112	+64



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Year	Quarter Q	Output Y	Moving total (4)	Moving total (8)	Moving Average (T)	Short-term Fluctuation Y-T=S+C+I
1973	1	47		957	120	-73
			485			
	2	158		1020	128	+30
			535			
	3	104		1084	136	-32
			549			
	4	226		1117	140	+86
			568			
1974	1	61		1166	146	-85
			598			
	2	177		1210	151	+26
			612			
	3	134		1235	154	-20
			623			
	4	240		1275	159	+81
			652			
1975	1	72		1311	164	-92
			659			
	2	206		1385	173	+33
			726			
	3	141				
	4	307				

Calculations of seasonal fluctuations :

Year	Q.1	Q.2	Q.3	Q.4
1971			-11	42
1972	-50	+12	-14	+64
1973	-73	+30	-32	+86
1974	-85	+26	-20	+81
1975	-92	+33		
Total	-300	+101	-77	+273
Average	-75	+25	-19	+68



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Other Methods of finding short period variations

- i. Simple Average : The method is very simple: average the data by months or quarters or years and then calculate the average for the period. Then find out, what percentage it is to the grand average.

$$\text{Seasonal Index} = \frac{\text{Monthly or quarterly average}}{\text{Grand Average of the months or the quarters}} \times 100$$

Same results are obtained if the totals of each month or each quarter are obtained instead of the average of each month or each quarter.

Illustration

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
1970	3.7	4.1	3.3	3.5
1971	3.7	3.9	3.6	3.5
1972	4.0	4.1	3.3	3.1
1973	3.3	4.4	4.0	4.1
Total	14.7	16.5	14.2	14.2
Quarterly Average	3.68	4.12	3.55	3.55
Seasonal Index	98.7	110.8	95.3	95.3

$$\text{The grand average} = \frac{3.68 + 4.12 + 3.55 + 3.55}{4} = \frac{14.90}{4} = 3.72$$

$$\text{Seasonal Index} = \frac{\text{Quarterly Average}}{\text{Grand Average}} \times 100$$

$$\text{Seasonal Index for the quarters : 1}^{\text{st}} : \frac{3.68}{3.72} \times 100 = 98.7$$

$$2^{\text{nd}} : \frac{4.12}{3.72} \times 100 = 110.8$$

$$3^{\text{rd}} \text{ and } 4^{\text{th}} : \frac{3.55}{3.72} \times 100 = 95.3$$

Example

Using the method of monthly averages determine the monthly indices for the following data of production of a commodity for the year 1979,1980,1981.



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Month	1979	1980	1981
Production in lacs of tons			
January	12	15	16
February	11	14	15
March	10	13	14
April	14	16	16
May	15	16	15
June	15	15	17
July	16	17	16
August	13	12	13
September	11	13	10
October	10	12	10
November	12	13	11
December	15	14	15

Solution

Here, instead of dividing the monthly average by the overall average, we divide the monthly totals by the average of totals.

Production in Lacks of tons

Year	1979	1980	1981	Seasonal Index	
Month				Totals	(percentages)
January	12	15	16	43	104.9
February	11	14	15	40	97.5
March	10	13	14	37	90.2
April	14	16	16	46	112.2
May	15	16	15	46	112.2
June	15	15	17	47	114.6
July	16	17	16	49	119.5
August	13	12	13	38	92.6
September	11	13	10	34	82.9
October	10	12	10	32	78
November	12	13	11	36	87.8
December	15	14	15	44	107.3
Total			492		
Average			41		



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Limitation: Although simple, this method is not very scientific for it assumes as if there is no trend component in the series, i.e., the original series comprise, $C \times S \times I$. Since most economic series have trends, the index computed by this method is actually an index of seasonal variation plus trend. Further, the effect of cycles on the original values may not be eliminated by the averaging process. It is only in case that the duration of the cycle coincides with the number of months or quarters included in the average that the cyclical fluctuations will be avoided. In the absence of this the seasonal index will also include the effect of trend.

(ii) Ratio-to-Trend Method : This method is an improvement over the previous method because this assumes that seasonal variation for a given month is a constant fraction of trend. This method presumably isolates the seasonal factor in the following manner :

$$S \times C \times I = \frac{T \times S \times C \times I}{T}$$

Random elements (I) are supposed to disappear when the ratios are averaged. Further, a carefully selected period of years used in computation is expected to eliminate the influence of cyclical fluctuations (C).

Example : Find seasonal variations by the ratio-to-trend method from the data given below.

Quarters

Year	1st	2nd	3rd	4th
1969	30	40	36	34
1970	34	52	50	44
1971	40	58	54	48
1972	54	76	68	62
1973	80	92	86	82

Solution

Firstly , we have to determine the trend value for each quarter as follows :

Year	Year Total	Quarterly Average	Deviation from mid-year	Trend Values			
Y	X	XY	X2				
1969	140	35	-2	-70	4	32	
1970	180	45	-1	-45	1	44	
1971	200	50	0	0	0	56	
1972	260	65	+1	+65	1	68	
1973	340	85	+2	+170	4	80	
		280		120	10		



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The value of the constants a and b in the equation of a straight line are as following :

$$a = \frac{\sum Y}{N} = \frac{280}{5} = 56, \quad a = \frac{\sum XY}{\sum X^2} = \frac{120}{10} = 12$$

The quarterly increment therefore is $12/4=3$.

Since there are 4 quarters we have to take the point in between the 2nd and the 3rd quarter each year. Thus following are trend values for various quarters of the respective years.

Year	1st	2nd	Yearly Trend	3rd	4th
Values					
1969	27.5	30.5	32	33.5	36.5
1970	39.5	42.5	44	45.5	48.5
1971	51.5	54.5	56	54.5	60.5
1972	63.5	66.5	68	69.5	72.5
1973	75.5	78.5	80	81.5	84.5

Actual Quarterly Values As % of quarterly Trend Values :					
1969	109.1	131.1		107.5	93.1
1970	86.1	122.4		109.9	90.7
1971	77.7	106.4		93.9	79.3
1972	85.0	114.3		97.8	85.5
1973	106.0	117.1		105.5	97.0
Total	463.9	591.3		514.6	445.6
Average	92.78	118.26		102.92	89.12
Seasonal Index					
Adjusted	92.0	117.4		102.1	88.4

Since the total of the seasonal index is $403.08 = (92.88+118.26+102.92+89.12)$.

Each index has to be adjusted by multiplying it by $\frac{400}{403.08}$ and the final indices are thus available.

(iii) Ratio-to-Moving Average Method : In this method instead of calculating the annual trend by the method of least squares the moving average is used. It is more suitable when seasonal variations for the months are to be calculated:



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Link Relative Method

(i) To Calculate Seasonal Link Relative for each season where

$$\text{Link Relative} = \frac{\text{Current Season Figure}}{\text{Previous Season Figure}} \times 100$$

- (ii) Calculate average of the link relatives for each season. Arithmetic mean is generally used but even median could be used.
- (iii) Convert these averages into chain relatives on the basis of first season.
- (iv) Calculate the Chain relative of the first season on the basis of the last season.
- (v) For correction, the chain relative of the first season calculated by the first method is deducted from the chain relative of the first season calculated by the second method. The difference is divided by the number of seasons. The resulting figure multiplied by 1,2,3, etc. are deducted respectively from the chain relatives of 2nd,3rd,4th, etc. seasons.
- (vi) The seasonal indices are available when we express the corrected chain relatives as percentages of their respective averages.

Example : Apply the method of link relatives to the following data and calculate seasonal indices.

Quarter	1969	1970	1971	1972	1973
I	4.5	4.8	4.9	5.2	6.0
II	5.4	5.6	6.3	6.5	7.0
III	7.2	6.3	7.0	7.5	8.4
IV	6.0	5.6	6.5	7.2	6.7

Solution : Let us first calculate link relatives of the seasonal figures.

These are given below in the table:

Year	Quarters			
	I	II	III	IV
1969		120	133	83
1970	80	117	113	89
1971	88	129	111	93
1972	80	125	115	96
1973	83	117	120	80
Mean	82.8	121.6	118.4	88



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Chain Relatives 100	$\frac{100 \times 121.6}{100}$	$\frac{121.6 \times 118.4}{100}$	$\frac{143.9 \times 88}{100}$
	= 121.6 = 143.9 = 126.6		
Corrected chain 100	121.6-1.2	143.9-2.4	126.6-3.6
	= 120.4 = 141.5 = 123		
Seasonal Indices	$\frac{100 \times 100}{121.2}$	$\frac{120.4 \times 100}{121.2}$	$\frac{141.5 \times 100}{121.2}$ $\frac{123.0 \times 100}{121.2}$
	= 82.5	= 99.4	= 116.7 = 101.5

Note : The link relatives are calculated by dividing the value of one quarter by the value of the previous quarter and expressing them as percentages.

For example, Link Relative of 1969-Quarter II = $\frac{5.4}{4.5} \times 100$
= 120

Link Relative of 1969-Quarter III = $\frac{7.2}{5.4} \times 100$
= 133

The figure correction has been calculated as follows :

Chain relative of the first quarter on the basis of first quarter = 100

Chain relative of first quarter on the basis on the last quarter = $\frac{82.8 \times 126.6}{100} = 104.8$

The difference between these chain relatives = 104.8 – 100 = 4.8

Difference per quarter $\frac{4.8}{4} = 1.2$

Seasonal variation indices have been calculated as follows

Average corrected chain relatives = $\frac{100 + 120.4 + 141.5 + 123.0}{4} = 121.2$

Seasonal variation indices therefore will be calculated as :

$\frac{\text{Corrected chain relative}}{121.2} = 100$



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Deseasonalization : The process of eliminating seasonal fluctuations or deseasonalization of data consists of dividing each value in the original series by the corresponding value of the seasonal index. The following table will illustrate this.

Quarter	Original Value	Seasonal Index	Deseasonal Value
I	272	79.000	344
II	381	120.153	317
III	346	109.583	315
IV	279	91.264	305

19.4 SMOOTHING METHODS IN TIME SERIES

Exponential Smoothing

The methods of analyzing the time series discussed earlier are usually employed for long term forecasting. But there are numerous situations e.g. , warehouse, consumer stores and manufacturing organizations which need future forecasts just over the next month, week or even a day, for each of the item individually.

It is not uncommon to find thousands and even lakhs of items in such organizations and, therefore, as many forecasts may have to be raised to maintain sufficient stocks of items. Excess stocking would adversely affect the service to the customer or hold up assembly of the final product.

Moving Averages method could indeed be applied in such situations, but it would require keeping in the records of the past sales figures as many in number as the moving average period. This would require too much clerical recording or in a computerized system, storage space in the computer files.

Exponential smoothing is a quicker and theoretically more sound method of short-term forecasting. Its major, though not exclusive, use, however, is in Production and Inventory control.

Exponential Smoothing as an Average

Consider the Moving Average (Period = 4 months) for the following series :

20 24 28 30

$$\begin{aligned}\text{Moving Average} &= \frac{1}{4} \times (20 + 24 + 28 + 30) \\ &= \frac{1}{4} \times 20 + \frac{1}{4} \times 24 + \frac{1}{4} \times 28 + \frac{1}{4} \times 30 \\ &= W_1 \times 20 + W_2 \times 24 + W_3 \times 28 + W_4 \times 30\end{aligned}$$



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It may be noted that the weight ages W_1, W_2, W_3 and W_4 assigned to each observation is equal and $\frac{1}{4}$ of that is reciprocal of the Moving Average period. Since the sum of the weight is unity the Moving Average is a true average.

Exponential Smoothing is also an average in this sense. But the weight ages assigned to the past figures are not equal. The most recent observation is assigned the highest weight age and it decreases in geometric as we move towards the older observations. This is obviously more reasonable because the most recent observation is more relevant for forecasting the future sales than an older observation. The ratio of the geometric progression of the series of the weights is $(1-\alpha)$, where α is a constant called smoothing coefficient. Theoretically all the past observations are needed for computing the exponentially smoothed average, that is the sequence of the past sales figures may run indefinitely to infinity. Despite this, as will be noted below, the method is remarkably easy to apply in practice.

The discussion should be more clear from the following table:

(The sum of the column C gives the exponentially smoothed average, u_t)

Observations at time t (A)	Weightage Assigned (B)	Col.(A) x Col (B) (C)
y_t	α	αy_t
y_{t-1}	$\alpha(1-\alpha)$	$\alpha(1-\alpha)y_{t-1}$
y_{t-2}	$\alpha(1-\alpha)^2$	$\alpha(1-\alpha)^2 y_{t-2}$
y_{t-3}	$\alpha(1-\alpha)^3$	$\alpha(1-\alpha)^3 y_{t-3}$
y_{t-n}	$\alpha(1-\alpha)^n$	$\alpha(1-\alpha)^n y_{t-n}$

Exp. Sm. Average. $u_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \dots + \alpha(1-\alpha)^n y_{t-n} + \dots$

This average may be used as the forecast for the next period, a in theory may be between 0 to 1: but in practice between 0.1 to 0.3.

It is to be contrasted with the moving average method in that infinite number of the past figures are being utilized, here. This looks awesome but a great deal of simplification is possible.

From Col. B it is obvious that the weights are decreasing exponentially $[\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \dots]$. Hence the name exponential smoothing.



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An essential feature of a true moving average is that the sum of the weights should be unity. Let us ascertain this for Exponential smoothing by summing up the column B.

$$\begin{aligned} & \alpha + \alpha(1-\alpha) + \dots + \alpha(1-\alpha)^n + \dots \\ & \alpha\{1 - (1-\alpha) + 1 - (1-\alpha)^2 + \dots\} \\ & = \alpha \left[\frac{1 - (1-\alpha)^n}{1 - (1-\alpha)} \right] = 1 - (1-\alpha)^n \end{aligned}$$

As n approaches ∞ , $(1-\alpha)$ being less than 1, approaches 0. Therefore, the sum of the weights is unity.

Procedure for Computations

$$\mu_1 = \alpha y_1 + \alpha(1-\alpha)y_{t-1} + \dots + \alpha(1-\alpha)^n y_{t-n} + \dots$$

$$\mu_1 = \alpha y_1 + (1-\alpha)\{\alpha y_{t-1} + \alpha(1-\alpha)y_{t-2} + \dots\}$$

$$\mu_1 = \alpha y_1 + (1-\alpha)\mu_{t-1}$$

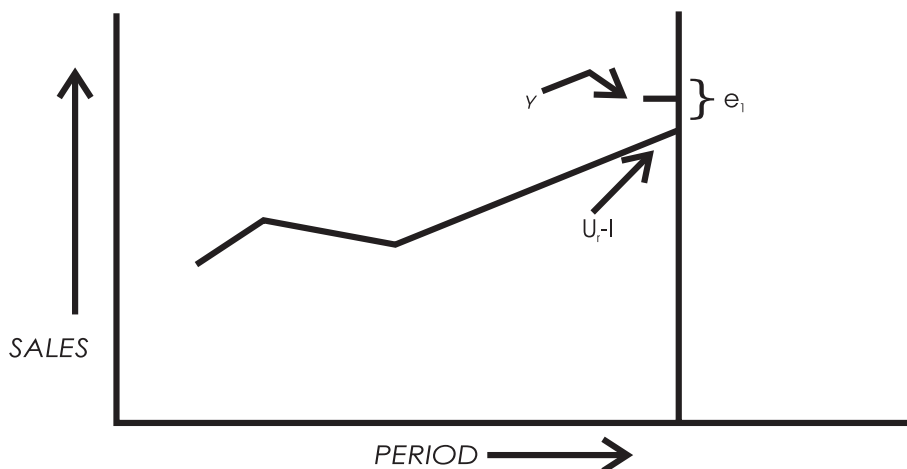
$$\mu_1 = \mu_{t-1} + \alpha(y_{t-1} - \mu_{t-1})$$

The term in the bracket on R.H.S> is the differences between latest observation and its forecast raised a period earlier. It is therefore, the error, and is denoted by e_1

$$\mu_1 = \mu_{t-1} + \alpha e_t$$

The procedure for computations is summarized below:

- (i) Find the error by subtracting the recent average from the incoming observation.
- (ii) Multiply error e^1 by α . This is the correction to be applied to the past average.
- (iii) Add the correction to the past coverage $\mu_1 = \mu_{t-1}$. This gives new average u_1 as the forecast for the next period. The rationale behind this is further clarified in the following figure:



Figure

The figure gives the forecasts prior to the period t . μ_{t-1} is the forecast for the period t raised at period $(t-1)$. The actual sales that materializes between the period $t-1$ and t are average y_t . The sales have exceeded the forecast. In view of this we should tend to modify the average. e_t is the error. The modification is done by increasing the average by e_t . Thus the difference between each new incoming observation and its forecast is being utilized in keeping the average up-to-date.

Value of the Smoothing Co-efficient

Consider the two extremes $\alpha = 0$ and $\alpha = 1$

- (i) $\alpha=0$
- (ii) The correction to be applied to the forecast is $\alpha=0$. This implies that the earlier average stands and no account of any difference between actual observation and its forecast is taken $\alpha=0$, therefore, implies that the forecasting system does not react to the incoming observations.
- (iii) $\mu_t = \mu_{t-1} + e_t = \mu_{t-1} + (y_t - \mu_{t-1}) = y_t$

Here the observation itself is taken as the forecast for the next period. The past series is completely ignored.

This should suggest that a low value of α gives more weightage to the past figures and less consideration to the incoming observation. Low values are therefore, to be used where the series is stable and high values where the series is rather fluctuating.

Eg:1. The actual figures for the sales of bleaching, powder for the year 1973 are shown below in Column A in the table below. The forecasts or the estimates using Exponential smoothing are required, $\alpha = 0.3$. Initial forecast may be taken as 29.6.



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Month	Sales (A)	Error (B) = (A) - (D)	Correction (C) = (B) x 0.3	Forecast (D)
Jan.	33	$33-29.6=3.4$	$3.4 \times 0.3 = 1.0$	29.6 (Initial)
Feb	31	$31-30.6=0.4$	$0.4 \times 0.3 = 0.1$	$29.6+1 = 30.6$
Mar	34	$34-30.7=3.3$	1	$30.6+0.1=30.7$
Apr	32	$32-31.7=0.3$	0.1	$30.7+1=31.7$
May	37	$37-31.8=5.2$	1.6	31.8
Jun	36	$36-33.4=2.6$	0.8	33.4
Jul	34	$34-34.2=-0.2$	-0.1	34.2
Aug	32	$32-34.1=-2.1$	-0.6	34.1
Sep	41	$41-33.5=7.5$	2.2	33.5
Oct	44	$44-35.7=8.3$	2.5	35.7
Nov	44	$44-38.2=5.8$	1.7	38.2
Dec	50	$50-39.9=10.1$	3.0	39.9

Note: (i) Each fig in col (d) is obtained by adding to its earlier figure the correction in column (c).

(ii) Initial estimate of 29.6 was given. When exponential smoothing is introduced this estimate would invariably be needed to start the system. It may be subjectively guessed or simple arithmetic mean of the past figures may be taken.

To summarise the discussion on Exponential smoothing thus far:

(A) Exponential smoothing, as against its contender the Moving Averages Method, is appealing because of the better weight age scheme.

(B) The greatest disadvantage inherent in the Moving Averages is relatively large amount of historical data that must be retained to compute them. The longer, the averaging period, the more the data that must be retained. In Exponential smoothing, records need to be kept just for α and the latest average.

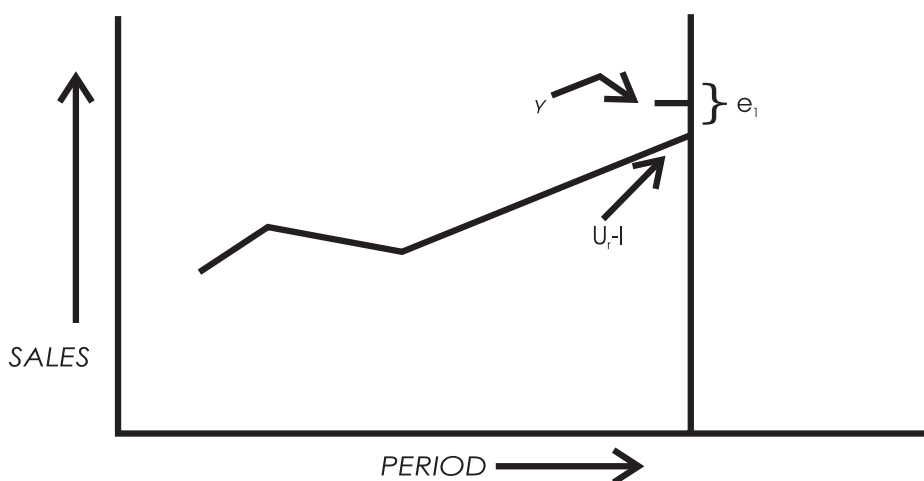
(C) Basically, Exponential smoothing provides a convenient, systematic method for revising the forecast for the next period whenever discrepancy exists between the previous forecast sales for the current period and the actual demand for the current period. If actual demand in the current period is higher than the forecast demand for the current period, the forecast for the next period should be adjusted upwards. The amount of adjustment is determined by the selected smoothing coefficient, the greater the constant the greater the adjustment and vice versa. In fact, if the behavior of the sales series changes say, from a stable one to a fluctuating one, may be changed to be responsive to the new behavior.



19.5 EXISTENCE OF TREND

The students might note in the solved example above by comparing column A with D that the forecast tends to be lower than the actual sales. This is because there exists a trend in the sales figures. The sales which is around 30 at commencement of the year 1973 have risen to 50 at the end.

The diagram below brings out the behaviour of the Exponentially smoothed average with regard to series depicting trend.



Figure

Thus the Exponential smoothed average lags the sales series with trend. The following equations [proof having been omitted] may be used, where trend is expected:

$$(i) u_t = u_{t-1} + \alpha e_t$$

$$(ii) \chi_t = \alpha(u_t - u_{t-1}) + (1-\alpha)\chi_{t-1}$$

$$(iii) u_t = u_t + \left(\frac{1-\alpha}{\alpha} + 1 \right) \chi_t$$

u_t gives the necessary forecast for the next period at time period t .



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Example 2 : Exponential Smoothing for series with Trend.

Assumed : Initial Trend = -3.00, Initial Average = 750.00, $\alpha = 0.1$

Period	Sales	Smoothed Average	Smoothed Trend	Forecast	Forecast Error
0		750	-3.00		
1	720	747	-3.00		
2	670	739.3	-3.47	717.0	47.00
3	680	733.37	-3.72	704.6	24.60
4	740	734.03	-3.28	696.2	-43.79
5	720	732.63	-3.09	701.3	-18.75
6	940	753.37	-0.71	701.7	-238.28
7	1020	780.03	2.03	746.3	-273.71
8	1220	824.03	6.23	800.3	-419.63
9	1260	867.62	9.96	886.3	-373.71
10	1300	910.86	13.29	967.3	-332.74
11	1190	938.78	14.75	1043.8	-146.23
12	1080	952.90	14.69	1086.3	6.30

Consider Period 1.

$$u_1 = u_o + \alpha e_1 = \alpha(y_1 - u_o) = 750 + 0.1(750 - 750) = 747$$

$$\lambda_1 = \alpha(u_1 - u_o) + (1 - \alpha)\lambda_o = 0.1(-3.00) + 0.9(-3.00) = -3.00$$

$$u'_1 = u_1 + \left(\frac{1 - \alpha}{\alpha} + 1 \right) \lambda = 747 + 10(-3.00) = 717$$

Forecast Error = 717 - 670 = 47. (Entered in the last column)

Consider Period 2

$$u_2 = u_1 + \alpha e_2 = u_1 + \alpha(y_2 - u_1) = 747 + 0.1(670 - 747) = 739.30$$

$$\lambda_2 = \alpha(u_2 - u_1) + (1 - \alpha)\lambda_1 = 0.1(739.30 - 747) + 0.9(-3.00) = -3.47$$

$$u_2 = u_2 + 10\lambda_2 = 739.30 - 34.70 = 704.60$$

Forecast Error. = 704.6 - 680 = 24.6

It is to be noted that the trend factor, λ_1 is also recursively exponentially smoothed for every period.



The student may carry on with the rest of calculation for practice by verifying with the figures in the table.

Optimum Value of the Smoothing Coefficient

In the above example, the forecast errors are given in the last column. These, when squared and accumulated, give a measure of the discrepancy between the forecasts and the actual sales. Similarly, other values of $\alpha = 0.2, 0.3$, etc. may be tried. The value that gives the least squared deviation of error would be optimum α . In practice, it is not infrequent to assign the value of α from experience. Where the computer activities are available, the above method of least squared deviations may be carried out for groups of items depicting similar sales behaviour.

Limitation of Exponential Smoothing

- (i) The method is useful for short-term forecasting only.
- (ii) It relies solely on the past history of sales. There are cases where subjective estimates may provide better forecasts. There have been attempts to complement the two; forecasts from Exponential smoothing and subjective estimates, successful to a great extent, but not fully satisfactory.

Further Extension

The method of Exponential smoothing is quite versatile and: trend can be extended to account for seasonal variations. Just as the initial, forecast and trend factors are estimated and then recursively adjusted in the light of the discrepancy between the forecast and the incoming observation; seasonal factors can be similarly initially estimated, incorporated in the model and then recursively adjusted. For short-term forecasting, it has almost universally superseded the Moving Average method. The method is particularly favoured with computerized applications because of the minimal data items that need to be stored.

19.6 FORECASTING USING TIME SERIES

Since economic and business conditions vary over time, business leaders must find ways to keep in touch with the effects that such change will have on their particular operations. One technique which business managers may use as an aid in planning for future needs is forecasting. There are basically two types of forecasting called qualitative and quantitative. Qualitative forecasting methods are useful when historical data are unavailable. Quantitative methods make use of historical data for forecasting. Quantitative forecasting methods can be subdivided into two types, namely, time series and causal. Causal forecasting methods involve the determination of factors which relate to the variable to be predicted. On the other hand, time series forecasting methods involve the projection of future values of a variable based entirely on the past and present observation of that variable. In this section we will see the various forecasting methods using time series.

- (i) Mean Forecast : The simplest forecasting method in which for the time period t we forecast the value of the series to be equal to the mean of the series;

$$y_t = \bar{y}$$



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This method is not adequate as trend effects and the cyclical effects are not taken into account in this.

- (ii) Naïve forecast : In this method, by taking advantage of the fact that there may be high correlation between successive pairs of values in a time series, we forecast the value, for the time period t, to-be equal to the actual value observed in the previous period t that is, time period (t – 1):

$$y_t = y_{t-1}$$

- (iii) Linear Trend Forecast : In this method , a linear relationship between the time and the response value has been found from the linear relationship.

$$y_t = a + bX$$

where X will be found from the value of t and a and b are constants.

- (iv) Non-linear Trend Forecast : In this method, a non-linear relationship between the time and the response value has been found again by least-squares method. Then the value, for the time period t , will be calculated from the non-linear equation . i.e.,

$$y_t = a + bX + cX^2$$

where X-value will be calculated from the value of t.

- (v) Forecasting will Exponential Smoothing : In this method, the forecast value for the time period t is found using exponential smoothing of time series. Specifically, at the time period t.

$$y_t = y_{t-1} + \alpha(y_t - y_{t-1})$$

where the forecasted value for time period t + 1 ;

y_{t+1} = the forecasted value for time period t. :

y_t = the observed value for time period t.

We will see the usefulness of these formulae with an example.

Example : The following time series data are given. Find the forecast for various years using mean forecast, naïve forecast, linear trend forecast, non-linear trend forecast.

Year	1980	1981	1982	1983	1984	1985
Sales (Rs. In Crores)	24.5	25.9	27.6	30.1	34.8	41.5

Solution

We list the various forecasting values in a table:

From the table, one can easily see that the observed and the forecast values using quadratic trend setting method do not differ much. Hence, here forecasting for future years can be done using quadratic trend setting. For example the forecast for 1991 will be Rs.48.69 crores. This is found from the quadratic equation $24.87 - 0.17 X + 0.69 X^2$ with base year



as 1985 i.e. $X = t - 1985$.

Table 2 : Comparison of from forecasting methods.

Year	Observed value y_t	Forecast value			
		Mean $y_t = y = 30.73$	Naïve $y_t = y_{t-1}$	Linear $y_t = 22.58 + 3.26 X$	Quadratic $y_t = 24.87 - 0.17X + 0.69X^2$
1985	24.5	30.73		22.58	24.87
1986	25.9	30.73	24.5	25.84	25.38
1987	27.6	30.73	25.9	29.1	24.27
1988	30.1	30.73	27.6	32.36	30.53
1989	34.8	30.73	30.1	35.63	35.17
1990	41.5	30.73	34.8	38.89	41.19

SUMMARY

- Utilities of time series analysis
 - ✓ Helps in evaluating current accomplishments
 - ✓ Helps in planning future operations
 - ✓ Facilitates comparisons
 - ✓ Helps in understanding past behaviour
- Components of time series
 - ✓ Secular trend (T)
 - ✓ Seasonal variations (S)
 - ✓ Cyclical variations (C)
 - ✓ Irregular variations (I)
- Methods for determining trend
 - ✓ Freehand
 - ✓ Semi average
 - ✓ Moving average
 - ✓ Least squares
- Methods for computing an index of seasonal variations
 - ✓ Simple average
 - ✓ Ratio to trends
 - ✓ Ratio to moving average
 - ✓ Link relative



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SELF-EXAMINATOIN QUESTIONS

1. Calculate the seasonal indices in the case of the following quarterly data in certain units assuming no trend in the data:

Year	Quarter			
	I	II	III	IV
1980	39	21	52	81
1981	45	23	63	76
1982	44	26	69	75
1983	53	26	64	84

Answer: 86.4, 44.4, 118.4 , 150.8)

2. The number of production in Bombay in from Quarters of a year during the period 1997- 1997 are given below:

Year	Quarter			
	I	II	III	IV
1987	165	135	140	180
1988	152	121	127	163
1989	44	26	69	75

Find the seasonal indices by Trend ratio method assuming a linear trend for the data.

Answer: 105, 83, 89, 123)

3. Compute Seasonal indices from the following data using method of Link Relatives.

Year	Quarter			
	I	II	III	IV
1978	65	58	56	61
1979	68	63	63	67
1980	70	59	56	52
1981	60	55	51	58

Answer: 109,98,94,99)

An old forecast of 39 is given initially and the smoothing co-efficient is 0.1. Smooth exponentially the following observations: 41,41,34,39,36,35,40,36,41,33.

Answer: 39.2, 39.38, 38.84, 38.86, 38.57, 38.21, 38.39, 38, 15, 38, 43, 37, 89)